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A new method for boarding passengers onto an airplane

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A B S T R A C T

We describe a new method to assign seats and to board passengers on an airplane that minimizes the total time to board. Steffen (2008) presents an optimum boarding method that assigns passengers to a specific numerical position in line that depends upon their ticketed seat location. Our method builds upon Steffen by assigning individual passengers to seats based on the amount of luggage they carry. Our heuristic method assigns passengers to seats so that their luggage is distributed evenly throughout the plane. Simulation results indicate that with our method, the total time to board all passengers on a fully loaded airplane is shorter than that of Steffen.

1. Introduction

In the past three decades, the airplane boarding process has become an increasingly important issue for airlines and passengers. Prior to 1970, the average boarding speed of passengers was approximately 20 passengers per minute. By 1998, this rate had decreased to approximately 9 passengers per minute (Marelli et al., 1998). The increased costs of checking luggage over the past decade will continue to play an important role in the time to board. If airlines continue to increase the fees for checked luggage, passengers are going to respond by carrying more luggage onto the plane. As a result, the boarding speed will continue to decrease. Nyquist and McFadden (2008) show that when using an average boarding strategy, the difference in boarding times when passengers have two carry-on bags compared to zero is almost 60%. Long boarding times impact costs. Ball et al. (2010) estimate the total cost of airline delays in 2007 was $29 billion in the United States alone. They separate this cost into three components: the cost to airlines ($8 billion), the cost to passengers ($17 billion), and the cost from lost demand ($4 billion). This indicates the possibility for large savings for the airlines and passengers with more efficient boarding methods.

Consistent with the current policy of many airlines, most previous publications assume that passengers are called to board a plane in groups (also known as blocks) and that within a group, passengers board in a random sequence (Bachmat and Elkin, 2008; Bazargan, 2007; Soolaki et al., 2012; Van den Briel et al., 2005). Van Landeghem and Beuselinck (2002) examine policies of passengers boarding by seat (i.e. passengers are called to board as individuals rather than by group). Their best policy tested has the passengers board in descending row order (i.e. fill the back of the plane first) and outside-in seat order (i.e. window seats are filled first, then middle seats, finally aisle seats). Audenaert et al. (2009) test a policy of boarding the fastest passengers first and another policy of boarding first the passengers with the most carry-on bags. The best of their policies (passengers with the most luggage board first) is about five percent better than boarding with assigned seats in a random (arbitrary) order in terms of the total time for all passengers to board. In contrast, the method of Steffen (2008) is about 24% faster than random (Steffen and Hotchkiss, 2012).

For ease of explanation, this paper assumes a 20 row airplane with six seats per row. To get to their seats, passengers walk down an aisle which separates the right side of the plane from the left. Referring to the airplane layout in Fig. 1, Steffen's model places the first passenger to board the plane in a window seat in the last (20th) row in the last (6th) column in the window seat labeled 1. The next passenger is also seated along the window, two rows in front of the first passenger in the seat labeled 2. This process continues for one side of the plane and then repeats on the other

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Entrance

Row	Window	Middle	Aisle		Aisle	Middle	Window
1	40		120				30
2	20						10
3	39		119				29
4	19						9
5	38		118				28
6	18						8
7	37		117				27
8	17						7
9	36		116				26
10	16						6
11	35		115				25
12	15						5
13	34		114				24
14	14						4
15	33		113				23
16	13					43	3
17	32		112				22
18	12					42	2
19	31		111				21
20	11					41	1

Fig. 1. Passengers boarding a plane in the sequence of the Steffen (2008) method.

side. The numbers under the window seats, middle seats, and aisle seats in Fig. 1 indicate the seats of the first 43 and final 10 passengers to board the plane with the last (120th) passenger to board assigned to the first aisle seat to his or her right after entering the plane. The result appears to be an optimal boarding sequence of passengers under the assumption that nothing is known in advance about the number of carry-on luggage of the individual passengers. In this paper, we leverage the method of Steffen, where the passengers' boarding sequence is determined by their seat assignment, but now add a novel means to assign each passenger to a seat based upon the number of pieces of luggage that they carry onto the airplane.

Although we use a single aisle cabin design in our paper, for our purposes, having two independent columns of passengers moving down two aisles of an airplane is not much different than having one column of passengers move down a single aisle. We anticipate that the main points of our paper apply regardless of whether an aircraft has one aisle or multiple aisles.

2. New method

The key aspect of our proposed method is that it assigns airplane passengers to seats so that their carry-on luggage is spread roughly evenly throughout the plane. This reduces the time passengers take to find available storage in the overhead bins when storing their luggage. We assume each passenger is carrying onto the plane either two bags, one bag, or zero bags which require storage in the overhead bin. In the first two steps of our method, we assign a set of two, one, or zero bags to the seats of the airplane without being concerned about which individual passenger carries that many bags. The third step of our method assigns an individual passenger carrying a specified number of bags to a particular seat designated through the first two steps as being allocated for someone carrying that specified number of bags. Finally, the fourth step of our method has passengers board according to the Steffen sequence based on their assigned seats. The key difference between our method and that of Steffen is that Steffen assumes passengers have been assigned to seats irrespective of the luggage they carry and our method assigns passengers to seats so that the luggage is distributed evenly throughout the plane. Below is our four-step procedure for how passengers should board an airplane.

Step 1: Assign sets of carry-on bags to rows.
Step 2: Within each row, assign sets of carry-on bags to seats.
Step 3: Assign passengers to seats matching the carry-on bags assignments from Step 2.
Step 4: Passengers board according to the Steffen sequence based on their assigned seats.

2.1. Step 1: assign sets of carry-on bags to rows

This step assigns two-bag, one-bag, and zero-bag passengers to each of the plane's rows to distribute the luggage throughout the length of the plane. This step also assigns the bags to the six columns of seats on the plane, beginning in column 1, but this column assignment is disregarded after this step. Each set of carry-on bags will be associated with a particular passenger during Step 3. For the discussion on the first two steps of our method, when we refer to a passenger being assigned to a row or a seat, we mean only that a particular quantity of luggage carried by some passenger will be allocated to the row or seat.

The row assignment is accomplished in Step 1 using a "greedy" heuristic by first assigning passengers carrying two bags to rows, then those carrying one bag, and finally the zero-bag passengers. (This greedy heuristic bears some resemblance to the "first fit decreasing" bin-packing algorithm described in Johnson, 1973). The two-bag passengers are placed into rows so that they are distributed approximately evenly across the length of the plane. Subsequently, one-bag passengers are assigned to rows which have as few bags as possible. Of these rows, the one-bag passengers are spread approximately evenly throughout length of the plane. Finally, if all seats are to be filled, then the passengers carrying zero bags are assigned to the remaining

Entrance

Row	Window	Middle	Aisle		Aisle	Middle	Window
1	2	1	1		1	0	0
2	2	1	1		1	0	0
3	2	1	1		1	0	0
4	2	2	1		0	0	0
5	2	1	1		1	0	0
6	2	1	1		1	0	0
7	2	2	1		1	0	0
8	2	1	1		1	0	0
9	2	1	1		1	0	0
10	2	1	1		1	0	0
11	2	2	1		0	0	0
12	2	1	1		1	0	0
13	2	1	1		1	0	0
14	2	2	1		1	0	0
15	2	1	1		1	0	0
16	2	1	1		1	0	0
17	2	1	1		1	0	0
18	2	2	1		0	0	0
19	2	1	1		1	0	0
20	2	1	1		1	0	0

Fig. 2. Each cell shows the number of bags carried by passengers in each seat at the completion of Step 1 of the algorithm.

seats; otherwise, the zero-bag passengers are assigned to those rows which have the fewest number of bags; among these rows, these passengers are spread approximately evenly throughout the length of the plane. Fig. 2 shows the results immediately after Step 1 from assigning 25 two-bag passengers, 52 one-bag passengers, and 43 zero-bag passengers. Observe that the passengers carrying two bags have all been assigned to the first (window) column and have been spread approximately evenly throughout the length of the plane. 18 of the rows have 5 bags and the two rows with 6 bags (rows 7 and 14) are about one third and two thirds of the plane's length from the entrance. See Appendix A for more details on how Step 1 of the method works.

2.2. Step 2: within each row, assign sets of carry-on bags to seats

Taking the row assignments of luggage from Step 1 as given, Step 2 spreads the luggage roughly evenly between the left and right sides of the aisle and leads toward passengers with more luggage tending to board the plane earlier than passengers with less luggage. Continuing to use the example row assignments of luggage in Fig. 2 the results of applying Step 2 of our method are shown in Fig. 3. Observe that the number of bags on either side of the plane in Fig. 2 is either two or three and tends to alternate. For example, on the left (right) side of the plane, rows 1 and 3 are allocated two (three) bags and rows 2 and 4 are allocated three (two) bags. Observe that passengers with the most bags will be assigned seats closest to the windows; because window seat passengers board the plane earliest under Steffen (Step 4). The window-seat passengers with two carry-on bags generally will be able to find spots in the relatively empty overhead bins without much difficulty. Step 2 proceeds in a greedy manner, row by row, assigning first the two-bag passengers to seats, secondly the one-bag passengers, and finally the zero-bag passengers. During this allocation, each passenger is assigned to whichever side of the plane has previously had the fewest number of bags assigned to it. Further details on Step 2 may be found in Appendix B.

Row	Window	Middle	Aisle	Entrance	Aisle	Middle	Window
1	1	1	0		0	1	2
2	2	1	0		0	1	1
3	1	1	0		0	1	2
4	2	1	0		0	0	2
5	1	1	0		0	1	2
6	2	1	0		0	1	1
7	2	1	0		0	1	2
8	1	1	0		0	1	2
9	2	1	0		0	1	1
10	1	1	0		0	1	2
11	2	1	0		0	0	2
12	1	1	0		0	1	2
13	2	1	0		0	1	1
14	2	1	0		0	1	2
15	1	1	0		0	1	2
16	2	1	0		0	1	1
17	1	1	0		0	1	2
18	2	1	0		0	0	2
19	1	1	0		0	1	2
20	2	1	0		0	1	1

Fig. 3. Each cell shows the number of bags carried by passengers in each seat at the completion of Step 2 of the algorithm.

2.3. Step 3: assign passengers to seats matching the carry-on bag assignments from step 2

The previous two steps determined the number of bags to be carried by a passenger sitting in each seat. Step 3 assigns arbitrarily an individual passenger to a seat carrying the number of bags specified at the end of Step 2 as belonging to a seat with that many bags allocated to it.

2.4. Step 4: passengers board according to the Steffen sequence based on their assigned seats

Finally, the fourth step of our method has passengers board according to the Steffen sequence based on their assigned seats. Whereas Steffen assigns passengers to seats arbitrarily, our proposed method assigns passengers to seats based on the luggage they carry using our steps 1–3. Once the passengers have been assigned to specific seats in our method, they board the plane in the Steffen sequence illustrated in Fig. 1.

3. Simulation model assumptions

We compare our method with that of Steffen using a discrete event simulation. Also we investigate the impact of spreading luggage throughout the plane when a sequence other than Steffen is used in Step 4 of our method. The parameters used are outlined in Table 1. We assume there are 20 rows in the plane and that each row has six passengers separated by a single aisle for a total of 120 seats for passengers on a full airplane. The row-to-row time represents the time it takes a passenger to move from one row of the plane to the next when walking down the aisle. The parameters and distributions of the row-to-row times (triangular distribution) match those of Van Landeghem and Beuselinck (2002) and Audenaert et al. (2009). The "time to sit without interference" parameter is the time it takes a passenger to sit after having arrived at his or her seat's row and having stored the luggage—assuming that no seated passengers in the row need to leave their seats in order for the current passenger to sit down. Van Landeghem and Beuslinck had gathered airplane boarding data from observations at Brussels National Airport; in their observations, a passenger sitting in an aisle seat often had to get up to allow another passenger to sit down in the same row's middle or window seat. The delays caused by such seat interferences are calculated explicitly in our model. Consequently, we use the minimum value of the time to sit parameters of Van Landeghem and Beuselinck (2002) as the minimum value of our "time to sit without interference." Since this minimum value is 3.33 times the minimum row-to-row time in Van Landeghem and Beuselinck (2002), we set all our time to sit without interference parameters to a value of 3.33 times the row-to-row parameters and like them we use the triangular distribution to model this. These values seem reasonable to us based on our personal experiences taking at least a few plane flights annually in recent

Table 1
Model parameters.

Parameter	Value
Plane characteristics {rows; seats per row; aisles}	{20; 6; 1}
Plane occupancy	100%
Time from one row to the next {min; mode; max} in seconds	{1.8; 2.4; 3}
Time to sit without interference {min; mode; max} in seconds	{6; 8; 10}

Table 2
Summary statistics for Steffen vs proposed method.

Probability customer carries			Time to board		
0 Bags	1 Bag	2 Bags	Steffen	Proposed	% Improvement
10%	60%	30%	8.02	7.84	2.3%
20%	50%	30%	7.97	7.76	2.6%
30%	50%	20%	7.72	7.49	3.0%
40%	40%	20%	7.66	7.43	3.0%
50%	40%	10%	7.41	7.28	1.7%
60%	30%	10%	7.35	7.22	1.8%
70%	20%	10%	7.30	7.19	1.5%
80%	10%	10%	7.24	7.16	1.1%

years. The time it takes a passenger to store luggage is modeled using Eq. (1) derived by Audenaert et al.

$$T_{\mathrm{sl}} = \left(\frac{n_{\mathrm{bin}} + n_p}{2} * n_p\right) * t_{\text{one row to next}} \tag{1}$$

The terms in Eq. (1) are defined as follows:

T_{sl} Time to store the luggage
n_{bin} The number of luggage already in the bin
n_p The number of luggage the passenger has
$t_{\text{one row to next}}$ Time from one row to the next from the parameters in Table 1

This model assumes there is one bin per row on each side of the plane and that each bin has unlimited storage space. The model accounts for the fact that as the bin fills with luggage, it takes longer to store additional luggage. Furthermore, it takes longer to store luggage as the number of bags the passenger has increases. One caveat is that Audenaert et al. did not provide an explicit derivation of their formula nor did they or we analyze data to support it.

On average, passengers who walk more slowly are likely to take more time to get into their seats. To reflect this for each passenger, our simulation model generates a single uniform random variable u with values between 0 and 1 and uses this same value of u in applying the inverse of the cumulative probability density function for each of the passenger's: time to move from one row to the next and time to sit without interference. For instance, a passenger with a randomly generated value of u equal to 0.99 would take about 3 s to move between rows and nearly 10 s to sit down in the absence of seat interference (maximum values for each). We assume a multinomial distribution for the number of bags carried by a passenger with probabilities of a passenger carrying two, one, or zero bags having values which vary depending on the test run.

We make the following assumptions on passenger flow. 1) Once boarding begins and continuing until the last passenger has begun moving down the aisle, there is always a passenger aboard the plane waiting at the aisle's entrance for the first row to become empty. 2) A passenger begins moving into the next row when it is clear of other passengers. 3) The row-to-row time is the time it takes a passenger to move from the beginning of a clear row to the end of that row. 4) A row becomes clear once all previous passengers have completed traversal of the row or have sat down in the row. 5) Passengers store luggage at the midpoint of a row and likewise sit at the midpoint of a row. Consequently, it takes a passenger half the row-to-row time to reach the middle of the row in which the passenger sits. 6) Once a passenger has reached the row containing his or her seat, the passenger stores any carry-on bags into the overhead bin above his or her seat. 7) Once any luggage has been stored, if the current passenger has a middle (window) seat, he or she instantaneously asks any passengers seated in the aisle (and middle) seats to move. Such seated passengers need to leave their seat (to make room for the current passenger) and re-sit after the current passenger has sat down. The time it takes such seated passengers to leave the seat and clear the row is the same as the time it took the passenger to sit down originally not counting seat interference. The time it takes for the passenger to re-sit is the same as the original time to sit not counting seat interference. Consequently, once the current passenger has stored luggage—if any—the total time to sit is the current passenger's time to sit without interference plus twice the time to sit without interference for each of the seated passengers who need to rise to make room for the current passenger.

4. Numerical results

Our simulation model was programmed in the Visual Basic programming language and invoked from Microsoft Excel. The simulations were run with 20,000 replications for each experimental condition tested. The same random number seed and thus the same set of random conditions (e.g. luggage and walking speed of each passenger) were used in the 20,000 replication runs for all experimental conditions. For the Table 2 experiments, we conducted a second set of runs for just the proposed method with 5000 replications and using a different random number seed; the 5000 replication results matched exactly those of the 20,000 replication results to the level of precision noted in the table, except for the final test case where a run time of 7.164 was rounded down to 7.16 for the 20,000 replication run and 7.166 was rounded up to 7.17 for the 5000 replication run.

Table 2 compares the mean times to board (in minutes) output by the simulation for our proposed method with that of the Steffen method as a function of the probability distribution values for the percentage of passengers carrying zero, one, and two bags requiring storage in the overhead bins. As indicated in the table, for all values of the multinomial probability distribution, our proposed method results in a total average time to finish boarding all passengers which is about one to three percent less than that of applying the Steffen method alone. Because of the various modeling assumptions made as described in the previous section, we caution the reader against interpreting the magnitude of the quantitative differences too precisely.

In addition, we investigated the impact of spreading luggage when passengers are called to board the plane in blocks of five rows each (e.g. the first passengers called to board the plane are those with seats in rows 16–20). Within each block passengers are assumed to board the plane in a random sequence. Table 3 contains the results of boarding by block under two conditions. In the first condition, "Luggage Spread Ignored," passengers are assigned to seats arbitrarily without respect to their number of carry-on bags. In the second condition, "Luggage Spread Considered," passengers are assigned seats based on their volume of carry-on luggage (using steps 1–3 of our proposed method). As indicated in Table 3, spreading the luggage throughout the plane results in slightly lower total times to board the plane when passengers are called to board the plane in blocks. Table 4 shows the results of passengers boarding the plane in a random sequence. The Table 4 results indicate that the "Luggage Spread Ignored" policy (using steps 1–3 of our proposed method) is

Table 3
Summary statistics for by block boarding sequence (with and without luggage spread).

Probability customer carries			Time to board		
			Luggage spread		
0 Bags	1 Bag	2 Bags	Ignored	Considered	% Improvement
10%	60%	30%	23.45	23.41	0.2%
20%	50%	30%	23.08	22.99	0.4%
30%	50%	20%	22.16	22.06	0.4%
40%	40%	20%	21.86	21.72	0.6%
50%	40%	10%	21.06	20.93	0.6%
60%	30%	10%	20.82	20.69	0.6%
70%	20%	10%	20.61	20.51	0.5%
80%	10%	10%	20.41	20.37	0.2%

Table 4
Summary statistics for random boarding sequence (with and without luggage spread).

Probability customer carries			Time to board		
			Luggage spread		
0 Bags	1 Bag	2 Bags	Ignored	Considered	% Improvement
10%	60%	30%	19.94	19.96	−0.1%
20%	50%	30%	18.75	18.74	0.0%
30%	50%	20%	18.08	18.07	0.1%
40%	40%	20%	17.87	17.82	0.3%
50%	40%	10%	17.30	17.23	0.4%
60%	30%	10%	17.13	17.06	0.4%
70%	20%	10%	16.97	16.93	0.2%
80%	10%	10%	16.83	16.83	0.0%

about the same as the policy of passengers boarding the plane in a random sequence irrespective of luggage. Consequently, spreading passengers across the length and width of the plane is most helpful when the Steffen sequence is used. This is fortunate because boarding times resulting from using Steffen for sequencing are superior to those resulting from block or random boarding.

5. Conclusions

For a fully loaded airplane, our method is about one to three percent faster than Steffen (2008) in terms of the total time required to board all passengers. For the experimental results of Table 2, our method averages a reduction of 0.16 min in boarding time compared with Steffen. Given that airlines incur a cost of about $30 per ground minute (Nyquist and McFadden, 2008) and Delta Airlines conducts 5800 flights per day (Delta Airlines, 2013), a reduction in boarding time of 0.16 min per flight would translate to a cost savings of about $10 million annually for a large airline such as Delta. We provide this $10 million figure only to provide an indication of the order of magnitude of the savings which may result from using our method at a large airline. Our calculations do not account for the varying types of aircraft and plane loads.

Both our method and Steffen require passengers to board the plane according to a pre-determined sequence reflecting their seat assignment. There are a number of ways this might be accomplished. One way is to print a boarding sequence number on the passenger's boarding pass. When a group of passengers possessing a set of consecutive sequence numbers is called to board, an airline employee can ensure the passengers line up according to their sequence numbers (Southwest Airlines, 2013 facilitates a similar process by having passengers line up next to columns which are labeled with relative boarding sequence numbers). Once this has become common practice, passengers would line themselves up in the proper sequence with only minimal instructions from the airline. Passengers approaching out of sequence would be instructed to move back in the line. As with all business processes, this change would require human adjustments. After passengers experience the faster boarding times resulting from our method (and reduced hassle in storing their luggage), we anticipate customer satisfaction would rise.

Our method also requires information (or at least an estimate) of the amount of luggage carried by each passenger. We believe this information can be obtained and that the airline industry has been moving in this direction. Some airlines charge fees for carry-on luggage. Spirit Airlines (2012) began charging $100 at the gate for passengers with carry-on luggage they did not pay to bring on board at the time of ticket purchase or check-in (one personal item carried on at no charge.) Alligiant Air (2013) charges a lower fee for carry-on luggage when paid on-line prior to arrival at the airport. Through these types of fees, airlines can encourage their passengers to provide accurate information on their carry-on luggage in advance of boarding. If an airline does not want to use fees for this purpose, the airline could estimate carry-on luggage from other known information. For example, a passenger departing Monday morning and returning the same day/night can be expected to have less carry-on luggage than a passenger departing that same Monday morning who is returning four days later without checking a bag.

Our method does not consider personal factors such as groups traveling together, passenger seat preferences, first class passengers, and passengers with special needs. These factors remain as consideration for further study. A general approach for addressing these factors may involve assigning families and high priority passengers to seats and boarding sequence position prior to assigning economy class passengers who purchase a single ticket.

Our paper illustrates the impact of passenger luggage on airplane boarding time. Space in the overhead bins is a precious resource and should be managed wisely. Spreading the luggage throughout the plane (with our method) is one aspect of managing the overhead bins resource. Airlines manage another precious resource (seats) through the dynamic setting of prices for seats on a flight as a function of the number of empty seats remaining, the number of days until takeoff, and other factors. Perhaps revenue management principles could be applied to set fees for carry-on luggage in an analogous manner to how the airlines maximize revenue through setting ticket fares. This opportunity remains for further study.

Appendix A

This appendix uses pseudocode to describe how Step 1 of our algorithm assigns sets of carry-on bags to rows so that the luggage is spread across the length of the plane.

```
Begin with an empty plane.
current_col ← 1
for NumBags ← maximum #bags carried by a passenger to 1 do
 while #unassigned passengers carrying NumBags > 0 do
  if #unassigned passengers carrying NumBags >= #unassigned rows in current_col then
      assign passengers carrying NumBags to all empty rows in current_col;
      current_col ← current_col + 1;
  else
      rows_to_assign ← rows with an open seat in the current_col
      min#bags ← fewest number of bags already assigned to a row in rows_to_assign
      Remove from rows_to_assign all rows with strictly more than min#bags assigned.
      if #unassigned passengers carrying NumBags >= # of rows_to_assign then
        assign the unassigned passengers carrying NumBags to these rows_to_assign
      else
        #seats_to_leave_empty ← # of rows_to_assign - #unassigned passengers carrying NumBags
        if #seats_to_leave_empty < #unassigned passengers carrying NumBags then
        #to_assign ← #seats_to_leave_empty
        assign = "empties"
      else
        #to_assign ← #unassigned passengers carrying NumBags;
        assign ← "passengers"
      #before_loop_to_assign ← #to_assign
      current_row ← 0
      c ← 1
      prev_row_assigned ← -1
      for p ← 1 to #before_loop_unassigned do
        ideal_row ←prev_row_assigned +((#rows on plane +1 – prev_row_assigned)
                                                    / (#to_assign + 1))
        while ideal_row > rows_to_assign(c) and c < # of rows_to_assign do
           c ← c + 1
        if c = 1 then
           row ← rows_to_assign(c)
        else
           if prev_row_assigned = rows_to_assign(c-1) then
```

```
    row ← rows_to_assign(c)
  else
    if ideal_row - rows_to_assign(c-1) >= rows_to_assign(c) - ideal_row
     and
      c+ # of unassigned rows_to_assign  <=  1 + #before_loop_unassigned then
            row ← rows_to_assign(c)
        else
            row ← rows_to_assign(c - 1)
            c ← c – 1
    if assign = "empties"
       empty_spot_assignment(p) ← row
    else
       assign NumBags to row in the current_col
       (This will reduce by one the:  #unassigned passengers carrying NumBags)
    prev_row_assigned ← row
    c ← c + 1
  if assign = "empties"  then
    e ←1
    for p ← 1 to #rows_to_assign  do
       if rows_to_assign(p) = empty_spot_assignment(e) then
         e ←e + 1
       else
         assign NumBags to rows_to_assign(p) in the current_col
         (This will reduce by one the:  #unassigned passengers carrying NumBags)
```

Appendix B

This appendix uses pseudocode to describe how Step 2 of our algorithm assigns sets of carry-on bags to seats within a row so that passengers with the most bags are seated in window seats and so that an equal or nearly equal number of bags are placed on each side of a row.

In the below algorithm, all passenger assignments to a side of a plane are made in outside-in seat order (i.e. window seats are assigned first, then middle seats, and finally aisle seats.) Furthermore, within a given row, the passengers are assigned in descending sequence of the number of bags they carry.

At this point, we know from Step 1 the #passengers to assign to each row and how many bags each carries.

```
Begin the assignment with an empty plane.
for row ← 1 to #rows on the plane do
  for p ← 1 to #passengers to be assigned to this row do
    if only one side of the plane has open seat(s) in this row then
      assign passenger p to that side of the plane in row which has open seat(s)
```

```
else
  if # of bags assigned to the left side of the plane across all rows <
    # of bags assigned to the right side of the plane across all rows then
    assign passenger p to the left side of the plane in row
  else
    if # of bags assigned to the left side of the plane across all rows >
      # of bags assigned to the right side of the plane across all rows then
      assign passenger p to the right side of the plane in row
    else
      if more bags have been assigned to the left side of row
        than have been assigned to the right side of row then
          assign passenger p to the right side of the plane in row
      else
        if more bags have been assigned to the right side of row
          than have been assigned to the left side of row then
            assign passenger p to the left side of the plane in row
        else
          assign passenger p to whichever side of the plane most recently
          in a row assignment had fewer bags than the other side of the
          plane in that row (or if there is no such row, assign passenger
          to either side of the plane arbitrarily)
```

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